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Lifetimes of Ξ_{bc}^{+} and Ξ_{bc}^{0} baryons

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Abstract. Estimates of the lifetimes and partial branching ratios for the baryons Ξ_{bc}^{+} and Ξ_{bc}^{0} are presented using the inverse heavy quark mass expansion technique carried out in the operator product expansion approach. We take into account both the perturbative QCD corrections to the spectator contributions and, depending on the quark contents of the hadrons, Pauli interference and weak scattering effects between the constituents, using a potential model for the evaluation of the non-perturbative parameters.

1 Introduction

Recently, techniques based on the inverse heavy quark mass ($1/m_Q$) expansion have been applied to study the properties of heavy hadrons in QCD [1–3]. These techniques are embedded in the operator product expansion (OPE) approach used in the context of effective theories. They allow one to calculate non-perturbative effects in the decays of heavy hadrons in terms of a few universal quantities, enabling one to extract the parameters of the standard model, such as the weak mixing angles involving quarks and heavy quark masses. The attained theoretical accuracy, reflecting the convergence of the series in both the inverse heavy quark mass and the QCD coupling constant, can be systematically improved and allows one to make precise predictions in the heavy quark sector in the standard model (SM), such as decay rates and distributions and partial rate asymmetries involving CP violation[1]. The virtual corrections due to "new" physics at a much higher scale may influence the decay characteristics of the heavy hadrons, enabling one to measure deviations from the SM-predictions in existing and forthcoming experiments.

The approach under discussion has been convincingly used in the description of weak decays of the hadrons having a single heavy quark, carried out in the framework of the heavy quark effective theory (HQET) [1], in the annihilation and radiative decays of heavy quarkonia $Q\bar{Q}$, using the framework of non-relativistic QCD (NRQCD) [2], and in the weak decays of long-lived heavy quarkonium with mixed heavy flavors B_c^+ [3][2]. In particular, we note that the experimental data on the weak decays of hadrons with two heavy quarks can be used to determine the parameters entering in the theoretical description of these systems. In turn, these parameters could also be determined in well-defined frameworks, say the potential model approach.

The baryons with two heavy quarks, $QQ'q$, provide new insight in the description of systems containing heavy quarks. For these baryons we can apply a method based on the combined HQET–NRQCD techniques [1–3], if we use the quark–diquark picture for the bound states. The expansion in the inverse heavy quark mass for the heavy diquark QQ' is a straightforward generalization of these techniques in the mesonic decays [2,3], with the difference that one is dealing, instead of the color-singlet systems, with the color–anti-triplet ones, with the appropriate account of the interaction with the light quark. First estimates of the lifetimes for the doubly charmed baryons Ξ_{cc} have recently been presented in [7]. The spectroscopic characteristics of baryons with two heavy quarks and the mechanisms of their production in different interactions have been discussed in [8,9], respectively.

In this paper, we present the calculation of lifetimes for the baryons Ξ_{bc}^{+} and Ξ_{bc}^{0}, containing the beauty and charmed quarks. As in the description of inclusive decays of the Ξ_{cc}-baryons, we follow the papers [10,3] where the necessary generalizations to the case of hadrons with two heavy quarks and other corrections are discussed. We note that keeping just the leading term in the OPE, the inclusive widths are determined by the mechanism of spectator decays involving free quarks, wherein the corrections in the perturbative QCD are taken into account. The inclusion of subleading terms in the expansion in the inverse heavy quarks mass[3] allows one to take into account the corrections due to the quark confinement inside the hadron. In this way, an essential role is played by the following non-perturbative characteristics: the motion of heavy quark inside the hadron and the corresponding time

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[1] For a review see [4]

[2] The first experimental observation of the B_c-meson has been reported by the CDF Collaboration [5]; see [6] for a review of the theoretical status of the B_c-meson decays

[3] It was shown in [11] that the leading order correction in $1/m_Q$ is absent and the corrections begin with $1/m_Q^2$

dilation in the hadron rest frame with respect to the quark rest frame, and the influence of the chromomagnetic interaction of the quarks. The important ingredient of such corrections in the baryons with two heavy quarks is the presence of a compact heavy diquark. The next peculiarity of baryons with two heavy quarks is the significant numerical influence on the lifetimes by the quark contents of hadrons, since in the third order in the inverse heavy quark mass, $1/m_Q^3$, the four-quark correlations in the total width are enforced in the effective lagrangian due to the two-particle phase space in the intermediate state (see the discussion in [10]). In this situation, we have to add the effects of the Pauli interference between the products of heavy quark decays and the quarks in the initial state as well as the scattering involving the quarks composing the hadron. Through such terms we introduce the corrections involving the masses of light and strange quarks in the framework of non-relativistic models with the constituent quarks. We include the corrections to the effective weak lagrangian due to the evolution of Wilson coefficients from the scale of the order of heavy quark mass to the energy, characterizing the binding of quarks inside the hadron.

This paper is organized as follows. Following the picture given above, we describe the general scheme for the construction of OPE for the total width of baryons with two heavy quarks with account of the corrections to the spectator width in Sect. 2. In Sect. 3, the procedure for the estimation of non-perturbative matrix elements of the states of doubly heavy baryons is considered for the operators of non-relativistic heavy quarks. Section 4 is devoted to the numerical evaluation of the lifetimes of Ξ_{bc}^{+} and Ξ_{bc}^{0} and of their partial decay rates, as well as to the discussion of the underlying uncertainties. We conclude in Sect. 5 by summarizing our results.

2 Operator product expansion for the heavy baryons

In accordance with the optical theorem, the total width $\Gamma_{\Xi_{bc}^{\diamond}}$ for the baryon $\Xi_{bc}^{\diamond}$, where $\diamond$ denotes the electrical charge, has the form

$$\Gamma_{\Xi_{bc}^{\diamond}} = \frac{1}{2M_{\Xi_{bc}^{\diamond}}}\langle \Xi_{bc}^{\diamond}|\mathcal{T}|\Xi_{bc}^{\diamond}\rangle, \tag{1}$$

where we accept the ordinary relativistic state normalization, $\langle \Xi_{bc}^{\diamond}|\Xi_{bc}^{\diamond}\rangle = 2EV$, and the transition operator $\mathcal{T}$:

$$\mathcal{T} = \Im m \int \mathrm{d}^4x\{\hat{T}H_{\mathrm{eff}}(x)H_{\mathrm{eff}}(0)\}, \tag{2}$$

is determined by the effective lagrangian of the weak interaction H_{eff} at the characteristic hadron energies:

$$H_{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{2\sqrt{2}}V_{q_2q_3}V^*_{Qq_1}[C_+(\mu)O_+ + C_-(\mu)O_-] + \mathrm{h.c.}, \tag{3}$$

where

$$O_{\pm} = [\bar{q}_{1\alpha}\gamma_\nu(1-\gamma_5)Q_\beta][\bar{q}_{2\gamma}\gamma^\nu(1-\gamma_5)q_{3\delta}]$$
$$\times(\delta_{\alpha\beta}\delta_{\gamma\delta} \pm \delta_{\alpha\delta}\delta_{\gamma\beta}),$$
$$C_+ = \left[\frac{\alpha_{\mathrm{s}}(M_W)}{\alpha_{\mathrm{s}}(\mu)}\right]^{6/(33-2f)},$$
$$C_- = \left[\frac{\alpha_{\mathrm{s}}(M_W)}{\alpha_{\mathrm{s}}(\mu)}\right]^{-12/(33-2f)},$$

so that f denotes the number of flavors, and Q marks the flavor of a heavy quark (b or c).

The quantity $\mathcal{T}$ in (1) permits one to use the operator product expansion in the inverse powers of heavy quark mass, which determines the energy release in the inclusive decays of baryon, containing the heavy quarks. Then the total width $\Gamma_{\Xi_{bc}^{\diamond}}$ contains the series in the non-perturbative matrix elements of operators with increasing dimensions in energy. In this way, for the subsystem composed of the heavy quarks only, we can additionally use the smallness of the relative velocity v in the movement of heavy quarks in contrast to the interaction with the light quark, wherein the only small order parameter is the ratio of the non-perturbative scale to the heavy quark mass $\Lambda_{\mathrm{QCD}}/m_Q$. Thus, there is the additional energy scale in the heavy subsystem along with the parameter given by the ratio of quark–gluon condensate scale to the heavy quark mass. It is the relative momentum of the quarks, m_Qv, which is much less than the energy release in the heavy quark decay, too.

Thus, OPE has the form

$$\mathcal{T} = \sum_{i=1}^{2}\left\{C_1(\mu)\bar{Q}^iQ^i + \frac{1}{m_{Q^i}^2}C_2(\mu)\bar{Q}^ig\sigma_{\mu\nu}G^{\mu\nu}Q^i + \frac{1}{m_{Q^i}^3}O(1)\right\}. \tag{4}$$

The leading contribution is given by the spectator decay, i.e. by the term $\bar{Q}Q$, which is the operator of dimension 3. The equations of motion allow one to show that there is no contribution by the operators with dimension 4. Further, there is only operator of dimension 5: $Q_{GQ} = \bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q$. Among the operators of dimension 6, $Q_{2Q2q} = \bar{Q}\Gamma q\bar{q}\Gamma' Q$, the dominant contributions are provided by Pauli interference and weak scattering. The latter ones are enforced by the two-particle phase space in comparison to the operators $Q_{61Q} = \bar{Q}\sigma_{\mu\nu}\gamma_l D^\mu G^{\nu l}Q$ and $Q_{62Q} = \bar{Q}D_\mu G^{\mu\nu}\Gamma_\nu Q$, which are neglected in what follows.

Then we have

$$\mathcal{T}_{\Xi_{bc}^{+}} = \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}_{6,\mathrm{PI}}^{(1)} + \mathcal{T}_{6,\mathrm{WS}}^{(1)},$$
$$\mathcal{T}_{\Xi_{bc}^{0}} = \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}_{6,\mathrm{PI}}^{(2)} + \mathcal{T}_{6,\mathrm{WS}}^{(2)},$$

where two initial terms denote the contributions into the decays of quark Q by the operators with the dimensions 3 and 5, and the emerging terms are the interference and scattering of constituents. In explicit form we find

$$\mathcal{T}_{35b} = \Gamma_{b,spec}\bar{b}b - \frac{\Gamma_{0b}}{m_b^2}[2P_{c1} + P_{c\tau1} + K_{0b}(P_{c1} + P_{cc1}) + K_{2b}(P_{c2} + P_{cc2})]O_{Gb}, \tag{5}$$

$$\mathcal{T}_{35c} = \Gamma_{c,\mathrm{spec}}\bar{c}c - \frac{\Gamma_{0c}}{m_c^2}[(2+K_{0c})P_{s1} + K_{2c}P_{s2}]O_{Gc}, \quad (6)$$

where

$$\Gamma_{0b} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2, \quad \Gamma_{0c} = \frac{G_F^2 m_c^5}{192\pi^3}, \quad (7)$$

with $K_{0Q} = C_-^2 + 2C_+^2, K_{2Q} = 2(C_+^2 - C_-^2)$ and $\Gamma_{Q,\mathrm{spec}}$ denotes the spectator width (see [11,13–15]):

$$P_{c1} = (1-y)^4, \quad P_{c2} = (1-y)^3, \quad (8)$$

$$P_{c\tau 1} = \sqrt{1-2(r+y)+(r-y)^2}[1-3(r+y) +3(r^2+y^2) - r^3 - y^3 - 4ry + 7ry(r+y)] + \Bigg(12r^2y^2 \times \ln\frac{(1-r-y+\sqrt{1-2(r+y)+(r-y)^2})^2}{4ry}\Bigg), \quad (9)$$

$$P_{cc1} = \sqrt{1-4y}(1-6y+2y^2+12y^3)24y^4 \times \ln\frac{1+\sqrt{1-4y}}{1-\sqrt{1-4y}}, \quad (10)$$

$$P_{cc2} = \sqrt{1-4y}\left(1+\frac{y}{2}+3y^2\right) - 3y(1-2y^2) \times \ln\frac{1+\sqrt{1-4y}}{1-\sqrt{1-4y}}, \quad (11)$$

where $y = m_c^2/m_b^2$ and $r = m_\tau^2/m_b^2$. The functions P_{s1} (P_{s2}) can be obtained from $P_{c1}(P_{c2})$ by the substitution $y = m_s^2/m_c^2$. In the b-quark decays, we neglect the value m_s^2/m_b^2 and suppose $m_s = 0$.

The calculation of both the Pauli interference effect for the products of heavy quark decays with the quarks in the initial state and the weak scattering of quarks composing the hadron, results in

$$\mathcal{T}_{6,\mathrm{PI}}^{(1)} = \mathcal{T}_{\mathrm{PI},u\bar{d}}^{c} + \mathcal{T}_{\mathrm{PI},s\bar{c}}^{b} + \mathcal{T}_{\mathrm{PI},d\bar{u}}^{b} + \sum_l \mathcal{T}_{\mathrm{PI},l\bar{\nu}_l}^{b}, \quad (12)$$

$$\mathcal{T}_{6,\mathrm{PI}}^{(2)} = \mathcal{T}_{\mathrm{PI},s\bar{c}}^{b} + \mathcal{T}_{\mathrm{PI},d\bar{u}}^{b} + \mathcal{T}_{\mathrm{PI},d\bar{u}}^{\prime b} + \sum_l \mathcal{T}_{\mathrm{PI},l\bar{\nu}_l}^{b}, \quad (13)$$

$$\mathcal{T}_{6,\mathrm{WS}}^{(1)} = \mathcal{T}_{\mathrm{WS},bu} + \mathcal{T}_{\mathrm{WS},bc}, \quad (14)$$

$$\mathcal{T}_{6,\mathrm{WS}}^{(2)} = \mathcal{T}_{\mathrm{WS},cd} + \mathcal{T}_{\mathrm{WS},bc}, \quad (15)$$

so that

$$\begin{aligned}
\mathcal{T}_{\mathrm{PI},s\bar{c}}^{b} &= -\frac{G_F^2|V_{cb}|^2}{4\pi}m_b^2\left(1-\frac{m_c}{m_b}\right)^2 \\
&\times\Bigg(\Bigg[\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4}\right) \\
&\times(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_i)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_j) \\
&+\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3}\right) \\
&\times(\bar{b}_i\gamma_\alpha\gamma_5 b_i)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_j)\Bigg] \quad (16) \\
&\times\left[(C_+ - C_-)^2 + \frac{1}{3}\left(1-k^{1/2}\right)(5C_+^2 + C_-^2 + 6C_-C_+)\right] \\
&+\Bigg[\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4}\right) \\
&\times(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_i) \\
&+\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3}\right) \\
&\times(\bar{b}_i\gamma_\alpha\gamma_5 b_j)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_i)\Bigg] \\
&\times k^{1/2}(5C_+^2 + C_-^2 + 6C_-C_+)\Bigg),
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{\mathrm{PI},\tau\bar{\nu}_\tau}^{b} &= -\frac{G_F^2|V_{cb}|^2}{\pi}m_b^2\left(1-\frac{m_c}{m_b}\right)^2 \\
&\times\Bigg[\left(\frac{(1-z_\tau)^2}{2} - \frac{(1-z_\tau)^3}{4}\right) \\
&\times(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_i) \\
&+\left(\frac{(1-z_\tau)^2}{2} - \frac{(1-z_\tau)^3}{3}\right) \\
&\times(\bar{b}_i\gamma_\alpha\gamma_5 b_j)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_i)\Bigg],
\end{aligned} \quad (17)$$

$$\begin{aligned}
\mathcal{T}_{\mathrm{PI},d\bar{u}}^{b'} &= -\frac{G_F^2|V_{cb}|^2}{4\pi}m_b^2\left(1-\frac{m_d}{m_b}\right)^2 \\
&\times\Bigg(\Bigg[\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4}\right) \\
&\times(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_i)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_j) \\
&+\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3}\right) \\
&\times(\bar{b}_i\gamma_\alpha\gamma_5 b_i)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_j)\Bigg] \quad (18) \\
&\times\left[(C_+ + C_-)^2 + \frac{1}{3}\left(1-k^{1/2}\right)(5C_+^2 + C_-^2 - 6C_-C_+)\right] \\
&+\Bigg[\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4}\right) \\
&\times(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_i) \\
&+\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3}\right) \\
&\times(\bar{b}_i\gamma_\alpha\gamma_5 b_j)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_i)\Bigg] \\
&\times k^{1/2}(5C_+^2 + C_-^2 - 6C_-C_+)\Bigg),
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{\mathrm{PI},u\bar{d}}^{c} &= -\frac{G_F^2}{4\pi}m_c^2\left(1-\frac{m_u}{m_c}\right)^2 \\
&\times\Bigg(\Bigg[\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4}\right) \\
&\times(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_j) \\
&+\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3}\right)
\end{aligned}$$

$$\left.\times(\bar{c}_i\gamma_\alpha\gamma_5 c_i)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_j)\right] \tag{19}$$

$$\times\left[(C_+ + C_-)^2 + \frac{1}{3}\left(1-k^{1/2}\right)\left(5C_+^2 + C_-^2 - 6C_-C_+\right)\right]$$

$$+\left[\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4}\right)\right.$$

$$\times(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_i)$$

$$+\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3}\right)$$

$$\left.\times(\bar{c}_i\gamma_\alpha\gamma_5 c_j)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_i)\right]$$

$$\left.\times k^{1/2}(5C_+^2 + C_-^2 - 6C_-C_+)\right),$$

$$\mathcal{T}_{\mathrm{WS},bc} = \frac{G_\mathrm{F}^2|V_{cb}|^2}{4\pi} m_b^2\left(1+\frac{m_c}{m_b}\right)^2 (1-z_+)^2$$

$$\times\left[\left(C_+^2 + C_-^2 + \frac{1}{3}\left(1-k^{1/2}\right)(C_+^2 - C_-^2)\right)\right.$$

$$\times(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_i)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_j) \tag{20}$$

$$\left.+k^{1/2}(C_+^2 - C_-^2)(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_i)\right],$$

$$\mathcal{T}_{\mathrm{WS},bu} = \frac{G_\mathrm{F}^2|V_{cb}|^2}{4\pi} m_b^2\left(1+\frac{m_u}{m_b}\right)^2 (1-z_+)^2$$

$$\times\left[\left(C_+^2 + C_-^2 + \frac{1}{3}\left(1-k^{1/2}\right)(C_+^2 - C_-^2)\right)\right.$$

$$\times(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_i)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_j) \tag{21}$$

$$\left.+k^{1/2}(C_+^2 - C_-^2)(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_i)\right],$$

$$\mathcal{T}_{\mathrm{WS},cd} = \frac{G_\mathrm{F}^2}{4\pi} m_c^2\left(1+\frac{m_d}{m_c}\right)^2 (1-z_+)^2$$

$$\times\left[\left(C_+^2 + C_-^2 + \frac{1}{3}\left(1-k^{1/2}\right)(C_+^2 - C_-^2)\right)\right.$$

$$\times(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_j) \tag{22}$$

$$\left.+k^{1/2}(C_+^2 - C_-^2)(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_i)\right],$$

$$\mathcal{T}_{\mathrm{PI},d\bar{u}}^{b} = \mathcal{T}_{\mathrm{PI},s\bar{c}}^{b}(z_- \to 0), \tag{23}$$

$$\mathcal{T}_{\mathrm{PI},e\bar{\nu}_e}^{b} = \mathcal{T}_{\mathrm{PI},\mu\bar{\nu}_\mu}^{b} = \mathcal{T}_{\mathrm{PI},\tau\bar{\nu}_\tau}^{b}(z_\tau \to 0), \tag{24}$$

where the following notation has been used:

$$\text{in(16)}\quad z_- = \frac{m_c^2}{(m_b-m_c)^2}, \quad k = \frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(m_b-m_c)},$$

$$\text{in(17)}\quad z_\tau = \frac{m_\tau^2}{(m_b-m_c)^2}, \quad k = \frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(m_b-m_c)},$$

$$\text{in(18)}\quad z_- = \frac{m_c^2}{(m_b-m_d)^2}, \quad k = \frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(m_b-m_d)},$$

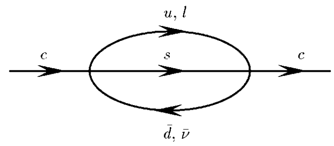

Fig. 1. The diagram of spectator contribution in the charmed quark decays

$$\text{in(19)}\quad z_- = \frac{m_s^2}{(m_c-m_u)^2}, \quad k = \frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(m_c-m_u)},$$

$$\text{in(20)}\quad z_+ = \frac{m_c^2}{(m_b+m_c)^2}, \quad k = \frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(m_b+m_c)},$$

$$\text{in(21)}\quad z_+ = \frac{m_c^2}{(m_b+m_u)^2}, \quad k = \frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(m_b+m_u)},$$

$$\text{in(22)}\quad z_+ = \frac{m_s^2}{(m_c+m_d)^2}, \quad k = \frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(m_c+m_d)}.$$

In the evolution of the coefficients C_+ and C_-, we have taken into account the threshold effects connected to the heavy quark masses.

In expressions (5) and (6), the scale μ has been taken to be approximately equal to m_c. In the Pauli interference term, we suggest that the scale can be determined on the basis of the agreement of the experimentally known difference between the lifetimes of Λ_c, Ξ_c^+ and Ξ_c^0 with the theoretical predictions in the framework described above[4]. In any case, the choice of the normalization scale leads to uncertainties in the final results. The theoretical accuracy can be improved by the calculation of next-order corrections in the powers of the QCD coupling constant.

The contributions by the leading terms $\bar{b}b$ and $\bar{c}c$, as they stand in (4), correspond to the imaginary parts of the diagrams like the one shown in Fig. 1. The coefficients for these operators are equivalent to the widths of the decays of free quarks and are known in the logarithmic approximation of QCD to the second order [16–20], including the mass corrections in the final state with the charmed quark and τ-lepton [20] in the decays of the b-quark and with the strange quark mass for the decays of the c-quark. To calculate the corrections to the logarithmic approximation, it is necessary to know the Wilson coefficients in the effective weak lagrangian to the next-to-leading order and the corrections with the single-gluon exchange in Fig. 1. In the numerical estimates, we include these corrections and mass effects, but we neglect the decay modes suppressed by the Cabibbo angle, and also the strange quark mass effects in b decays. The bulky expressions for the spectator widths are given in the Appendix of [7].

The $\sum_{i=1}^{2} O_{GQ^i}$ contributions are obtained by the calculation of diagrams like the one in Fig. 1 with all possible insertions of the external gluon attached to the internal quark lines. The corresponding expressions are known in the logarithmic approximation. Finally, the contributions by the operators with dimension 6 are obtained by cutting a single internal quark line in Fig. 1, which results in the diagrams of Fig. 2. These terms correspond to the

[4] A more expanded description is presented in [7]

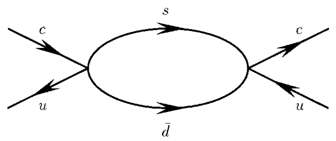

Fig. 2. The diagram for the contribution of Pauli interference in the decays of the charmed quark for the Ξ_{bc}^{+} baryon

Pauli interference of decay products with the quarks from the initial state, and the weak scattering of the quarks composing the hadron. We have calculated these effects with account of the masses in the final states and for the logarithmic renormalization of the effective lagrangian for the non-relativistic heavy quarks at energies less than the heavy quark masses.

To calculate the semileptonic decay branching fraction for the baryon $\Xi_{bc}^{\diamond}$, we have used the expressions from [14, 20]:

$$\begin{aligned}\Gamma_{sl} = \sum_{i=1}^{2} 4\Gamma_{Q^i}(&\{1 - 8\rho^i + 8\rho^{i3} - \rho^{i4} - 12\rho^{i2}\ln\rho^i\} \\ &+ E_{Q^i}\{5 - 24\rho^i + 24\rho^{i2} - 8\rho^{i3} + 3\rho^{i4} - 12\rho^{i2}\ln\rho^i\} \\ &+ K_{Q^i}\{-6 + 32\rho^i - 24\rho^{i2} - 2\rho^{i4} + 24\rho^{i2}\ln\rho^i\} \\ &+ G_Q^i\{-2 + 16\rho^i - 16\rho^{i3} + 2\rho^{i4} + 24\rho^{i2}\ln\rho^i\}), \quad (25)\end{aligned}$$

where

$$\Gamma_c = |V_{cs}|^2 G_{\mathrm{F}}^2 \frac{m_c^5}{192\pi^3}, \quad \Gamma_b = |V_{cb}|^2 G_{\mathrm{F}}^2 \frac{m_b^5}{192\pi^3}, \qquad (26)$$

$$\rho^1 = \frac{m_s^2}{m_c^2}, \quad \rho^2 = \frac{m_c^2}{m_b^2}. \qquad (27)$$

The quantities $E_{Q^i} = K_{Q^i} + G_{Q^i}$, K_{Q^i} and G_{Q^i} are determined by the formulae

$$\begin{aligned}K_{Q^i} &= -\langle \Xi_{bc}^{\diamond}(v)|\bar{Q}_v^i \frac{(\mathrm{i}D)^2}{2m_{Q^i}^2} Q_v^i|\Xi_{bc}^{\diamond}(v)\rangle, \\ G_{Q^i} &= \langle \Xi_{bc}^{\diamond}(v)|\bar{Q}_v^i \frac{gG_{\alpha\beta}\sigma^{\alpha\beta}}{4m_{Q^i}^2} Q_v^i|\Xi_{bc}^{(*)}(v)\rangle, \qquad (28)\end{aligned}$$

whereas the spinor field of the effective theory Q_v^i is given by the form

$$Q^i(x) = \mathrm{e}^{-\mathrm{i}m_{Q^i}v\cdot x}\left[1 + \frac{\mathrm{i}\not{D}}{m_{Q^i}}\right] Q_v^i(x). \qquad (29)$$

We have also taken into account the known α_{s}-corrections to the semileptonic quark decay width.

Thus, the calculation of the lifetime for the baryon $\Xi_{bc}^{\diamond}$ is reduced to the problem of the evaluation of the matrix elements of the operators, which is the subject of the next section.

3 Hadronic matrix elements

In accordance with the equations of motion, the matrix element of the operator $\bar{Q}^j Q^j$ can be expanded in a series in powers of $1/m_{Q^j}$:

$$\begin{aligned}&\langle \Xi_{bc}^{\diamond}|\bar{Q}^j Q^j|\Xi_{bc}^{\diamond}\rangle_{\mathrm{norm}} = 1 \qquad (30)\\ &-\frac{\langle \Xi_{bc}^{\diamond}|\bar{Q}^j\left[(\mathrm{i}\boldsymbol{D})^2 - \left(\frac{\mathrm{i}}{2}\sigma G\right)\right]Q^j|\Xi_{bc}^{\diamond}\rangle_{\mathrm{norm}}}{2m_{Q^j}^2} + O\left(\frac{1}{m_{Q^j}^3}\right).\end{aligned}$$

So, we have to estimate the numerical values involving the following set of operators:

$$\begin{aligned}&\bar{Q}^j(\mathrm{i}\boldsymbol{D})^2 Q^j, \quad \left(\frac{\mathrm{i}}{2}\right)\bar{Q}^j\sigma G Q^j, \\ &\bar{Q}^j\gamma_\alpha(1-\gamma_5)Q^j\bar{q}\gamma^\alpha(1-\gamma_5)q, \\ &\bar{Q}^j\gamma_\alpha\gamma_5 Q^j\bar{q}\gamma^\alpha(1-\gamma_5)q, \quad \bar{Q}^j\gamma_\alpha\gamma_5 Q^j\bar{Q}^k\gamma^\alpha(1-\gamma_5)Q^k, \\ &\bar{Q}^j\gamma_\alpha(1-\gamma_5)Q^j\bar{Q}^k\gamma^\alpha(1-\gamma_5)Q^k. \qquad (31)\end{aligned}$$

The first term corresponds to the motion of the quark inside the hadron, and it results in the corrections caused by the time dilation in the hadron rest frame with respect to the quark rest frame. The second term introduces the corrections due to the chromomagnetic interaction of quarks. The third and fourth terms depend on the four-quark fields and are connected with the contributions from the Pauli interference and weak scattering.

Further, following the general approach of effective theories, we introduce the effective field Ψ_Q, which, for the case under consideration, represents the non-relativistic spinor of heavy quark, so that we account for the virtualities μ in the rage of $m_Q > \mu > m_Q v_Q$ in the framework of perturbative QCD. The non-perturbative effects in the matrix elements have to be expressed in terms of effective non-relativistic fields. So we have

$$\begin{aligned}\bar{Q}Q = \Psi_Q^\dagger\Psi_Q &- \frac{1}{2m_Q^2}\Psi_Q^\dagger(\mathrm{i}\boldsymbol{D})^2\Psi_Q + \frac{3}{8m_Q^4}\Psi_Q^\dagger(\mathrm{i}\boldsymbol{D})^4\Psi_Q \\ &- \frac{1}{2m_Q^2}\Psi_Q^\dagger g\boldsymbol{\sigma}\boldsymbol{B}\Psi_Q - \frac{1}{4m_Q^3}\Psi_Q^\dagger(\boldsymbol{D}g\boldsymbol{E})\Psi_Q + \cdots \quad (32)\end{aligned}$$

$$\bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q = -2\Psi_Q^\dagger g\boldsymbol{\sigma}\boldsymbol{B}\Psi_Q - \frac{1}{m_Q}\Psi_Q^\dagger(\boldsymbol{D}g\boldsymbol{E})\Psi_Q + \cdots \qquad (33)$$

We omit the term of $\Psi_Q^\dagger\boldsymbol{\sigma}(g\boldsymbol{E}\times\boldsymbol{D})\Psi_Q$, corresponding to the spin–orbital interactions, which are not essential for the ground state of the baryons under consideration. For the normalization, we suppose

$$\int \mathrm{d}^3x\Psi_Q^\dagger\Psi_Q = \int \mathrm{d}^3x Q^\dagger Q. \qquad (34)$$

Then, for Q defined as

$$Q \equiv \mathrm{e}^{-\mathrm{i}mt}\begin{pmatrix}\phi \\ \chi\end{pmatrix}, \qquad (35)$$

we find

$$\Psi_Q = \left(1 + \frac{(\mathrm{i}\boldsymbol{D})^2}{8m_c^2}\right)\phi. \qquad (36)$$

Let us stress the difference between the descriptions for the interaction of a heavy quark with the light one and of a heavy quark with the heavy one. As we have already mentioned above, in the heavy subsystem there is an additional parameter which is the relative velocity of the quark. It introduces the energy scale equal to $m_Q v$. Therefore, the Darwin term $(\boldsymbol{DE})$ in the heavy subsystem stands in the same order in the inverse heavy quark mass in comparison to the chromomagnetic term $(\boldsymbol{\sigma B})$ (they have the same power in the velocity v). This statement becomes evident if we apply the scaling rules in NRQCD [12]:

$$\Psi_Q \sim (m_Q v_Q)^{3/2}, \quad |\boldsymbol{D}| \sim m_Q v_Q, \quad gE \sim m_Q^2 v_Q^3,$$

$$gB \sim m_Q^2 v_Q^4, \quad g \sim v_Q^{1/2}.$$

For the interaction of the heavy quark with the light one, there is no such small velocity parameter, so that the Darwin term is suppressed by the additional factor of $k/m_Q \sim \Lambda_{\mathrm{QCD}}/m_Q$.

Further, experience of the phenomenology in the potential quark models shows that the kinetic energy of quarks practically does not depend on the quark contents of the system, and it is determined by the color structure of the state. So, we suppose that the kinetic energy is equal to $T = m_d v_d^2/2 + m_l v_l^2/2$ for the quark–diquark system, and it is $T/2 = m_b v_b^2/2 + m_c v_c^2/2$ in the diquark case (note the color factor of 1/2). Then

$$\frac{\langle \Xi_{bc}^\diamond | \Psi_c^\dagger (\mathrm{i}\boldsymbol{D})^2 \Psi_c | \Xi_{bc}^\diamond \rangle}{2M_{\Xi_{bc}^\diamond} m_c^2} \simeq v_c^2 \simeq \frac{2m_l T}{(m_l+m_b+m_c)(m_b+m_c)} + \frac{m_b T}{m_c(m_c+m_b)}, \quad (37)$$

$$\frac{\langle \Xi_{bc}^\diamond | \Psi_b^\dagger (\mathrm{i}\boldsymbol{D})^2 \Psi_b | \Xi_{bc}^\diamond \rangle}{2M_{\Xi_{bc}^\diamond} m_b^2} \simeq v_b^2 \simeq \frac{2m_l T}{(m_l+m_b+m_c)(m_b+m_c)} + \frac{m_c T}{m_b(m_c+m_b)}. \quad (38)$$

Numerically, we assign $T \simeq 0.4\,\mathrm{GeV}$, which results in $v_c^2 = 0.195$ and $v_b^2 = 0.024$, where the dominant contribution is provided by the motion inside the diquark.

Define

$$O_{\mathrm{mag}} \equiv \frac{g_s}{4m_c}\bar{c}\sigma_{\mu\nu}G^{\mu\nu}c + \frac{g_s}{4m_b}\bar{b}\sigma_{\mu\nu}G^{\mu\nu}b, \quad (39)$$

$$O_{\mathrm{mag}} = \frac{\lambda}{m_c}(S_{cl}(S_{cl}+1) - S_c(S_c+1) - S_l(S_l+1)) + \frac{\lambda}{m_b}(S_{bl}(S_{bl}+1) - S_b(S_b+1) - S_l(S_l+1)), \quad (40)$$

where $S_{bl} = S_b + S_l$, $S_{cl} = S_c + S_l$, S_b is the b-quark spin, S_c is that of the c-quark, and S_l is the light quark spin. The operator under study is related to the hyperfine splitting in the baryon system:

$$\begin{aligned} &\langle S_{bc}=1, S=\tfrac{3}{2}|O_{\mathrm{mag}}|S_{bc}=1, S=\tfrac{3}{2}\rangle \\ &-\langle S_{bc}=1, S=\tfrac{1}{2}|O_{\mathrm{mag}}|S_{bc}=1, S=\tfrac{1}{2}\rangle \\ &= \langle S_{bc}=1, S=\tfrac{3}{2}|V_{hf}|S_{bc}=1, S=\tfrac{3}{2}\rangle \\ &-\langle S_{bc}=1, S=\tfrac{1}{2}|V_{hf}|S_{bc}=1, S=\tfrac{1}{2}\rangle, \end{aligned} \quad (41)$$

so that S denotes the total spin of the system, and S_{bc} is the diquark spin. Further, the perturbative term, depending on the spins, is equal to

$$V_{hf} = \frac{8}{9}\alpha_{\mathrm{s}}\frac{1}{m_l m_c}\mathbf{S}_l\mathbf{S}_c|R^{dl}(0)|^2 + \frac{8}{9}\alpha_{\mathrm{s}}\frac{1}{m_l m_b}\mathbf{S}_l\mathbf{S}_b|R^{dl}(0)|^2, \quad (42)$$

where $R^{dl}(0)$ is the radial wave function at the origin of the quark–diquark system. In contrast to the diquark system with the identical quarks, this operator is not diagonal in the basis of S and S_{bc}. To proceed, we use

$$|S;S_{bc}\rangle = \sum_{S_{bl}}(-1)^{(S+S_l+S_c+S_b)}\sqrt{(2S_{bl}+1)(2S_{bc}+1)} \times \left\{\begin{matrix} S_l & S_b & S_{bl} \\ S_c & S & S_{bc}\end{matrix}\right\}|S;S_{bl}\rangle \quad (43)$$

and

$$|S;S_{bc}\rangle = \sum_{S_{cl}}(-1)^{(S+S_l+S_c+S_b)}\sqrt{(2S_{cl}+1)(2S_{bc}+1)} \times \left\{\begin{matrix} S_l & S_c & S_{cl} \\ S_b & S & S_{bc}\end{matrix}\right\}|S;S_{cl}\rangle. \quad (44)$$

The result of all substitutions is

$$\lambda = \frac{4|R^{dl}(0)|^2\alpha_{\mathrm{s}}}{9m_l}, \quad (45)$$

which can be used in the calculations for the baryon with the vector diquark; however, in what follows we put $S_{bc} = 0$, so that

$$\frac{\langle \Xi_{bc}^\diamond | O_{\mathrm{mag}} | \Xi_{bc}^\diamond \rangle}{2M_{\Xi_{bc}^\diamond}} = 0. \quad (46)$$

The account of Darwin and chromomagnetic terms results in

$$\frac{\langle \Xi_{bc}^\diamond | \Psi_c^\dagger g\boldsymbol{\sigma}\cdot\boldsymbol{B}\Psi_c | \Xi_{bc}^\diamond \rangle}{2M_{\Xi_{bc}^\diamond}} = -\frac{2}{3}g^2\frac{|\Psi^d(0)|^2}{m_b}, \quad (47)$$

$$\frac{\langle \Xi_{bc}^\diamond | \Psi_c^\dagger (\boldsymbol{D}\cdot g\boldsymbol{E})\Psi_c | \Xi_{bc}^\diamond \rangle}{2M_{\Xi_{bc}^\diamond}} = \frac{2}{3}g^2|\Psi^d(0)|^2, \quad (48)$$

where $\Psi^d(0)$ is the wave function at the origin of the diquark. The analogous matrix elements[5] for the operators

[5] The obtained expressions differ from those for the B_c-meson [3] because of the color structure of the state, providing the factor of 1/2

of the beauty quarks can be written down by a permutation of the heavy quark masses.

Combining the results above, we find

$$\frac{\langle \Xi_{bc}^{\diamond}|\bar{c}c|\Xi_{bc}^{\diamond}\rangle}{2M_{\Xi_{bc}^{\diamond}}} = 1 - \frac{1}{2}v_c^2 + \frac{g^2}{3m_b m_c^2}|\Psi^d(0)|^2 - \frac{1}{6m_c^3}g^2|\Psi^d(0)|^2 + \dots \approx 1 - 0.097 + 0.004 - 0.007\dots \quad (49)$$

The dominant role in the corrections is played by the term connected to the time dilation because of the quark motion inside the diquark. Next, for the operator $cg\sigma_{\mu\nu}G^{\mu\nu}c$ we have

$$\frac{\langle \Xi_{bc}^{\diamond}|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Xi_{bc}^{\diamond}\rangle}{2M_{\Xi_{bc}^{\diamond}}m_c^2} = \frac{4g^2}{3m_b m_c^2}|\Psi^d(0)|^2 - \frac{g^2}{3m_c^3}|\Psi^d(0)|^2 \approx 0.002. \quad (50)$$

The permutation of quark masses leads to the required expressions for the operators of $\bar{b}b$ and $\bar{b}g\sigma_{\mu\nu}G^{\mu\nu}b$.

For the four quark operators, determining the Pauli interference and the weak scattering, we use the estimates in the framework of the non-relativistic potential model [7]:

$$(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{c}\gamma^\mu(1-\gamma_5)c) = 2(m_c+m_b)|\Psi^d(0)|^2 \times(1-4S_bS_c), \quad (51)$$
$$(\bar{b}\gamma_\mu\gamma_5 b)(\bar{c}\gamma^\mu(1-\gamma_5)c) = -4S_bS_c\cdot 2(m_c+m_b) \times|\Psi^d(0)|^2, \quad (52)$$
$$(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{q}\gamma^\mu(1-\gamma_5)q) = 2(m_b+m_l)|\Psi^{dl}(0)|^2 \times(1-4S_bS_q), \quad (53)$$
$$(\bar{b}\gamma_\mu\gamma_5 b)(\bar{q}\gamma^\mu(1-\gamma_5)q) = -4S_bS_q\cdot 2(m_b+m_l) \times|\Psi^{dl}(0)|^2, \quad (54)$$
$$(\bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{q}\gamma^\mu(1-\gamma_5)q) = 2(m_c+m_l)|\Psi^{dl}(0)|^2 \times(1-4S_cS_q), \quad (55)$$
$$(\bar{c}\gamma_\mu\gamma_5 c)(\bar{q}\gamma^\mu(1-\gamma_5)q) = -4S_cS_q\cdot 2(m_c+m_l) \times|\Psi^{dl}(0)|^2. \quad (56)$$

The exploitation of (43) and (44) for the basic states of the baryons results in

$$\langle \Xi_{bc}^{\diamond}|(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{c}\gamma^\mu(1-\gamma_5)c)|\Xi_{bc}^{\diamond}\rangle = 8(m_b+m_c) \times|\Psi^d(0)|^2, \quad (57)$$
$$\langle \Xi_{bc}^{\diamond}|(\bar{b}\gamma_\mu\gamma_5 b)(\bar{c}\gamma^\mu(1-\gamma_5)c)|\Xi_{bc}^{\diamond}\rangle = 6(m_b+m_c) \times|\Psi^d(0)|^2, \quad (58)$$
$$\langle \Xi_{bc}^{\diamond}|(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bc}^{\diamond}\rangle = 2(m_b+m_l) \times|\Psi^{dl}(0)|^2, \quad (59)$$
$$\langle \Xi_{bc}^{\diamond}|(\bar{b}\gamma_\mu\gamma_5 b)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bc}^{\diamond}\rangle = 0, \quad (60)$$
$$\langle \Xi_{bc}^{\diamond}|(\bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bc}^{\diamond}\rangle = 2(m_c+m_l) \times|\Psi^{dl}(0)|^2, \quad (61)$$
$$\langle \Xi_{bc}^{\diamond}|(\bar{c}\gamma_\mu\gamma_5 c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bc}^{\diamond}\rangle = 0. \quad (62)$$

Table 1. The widths of the inclusive spectator decays for the b- and c-quarks, in ps^{-1}

Mode	$b\to c\bar{u}d$	$b\to c\bar{c}s$	$b\to ce^+\nu$
Γ	0.310	0.137	0.075
Mode	$b\to c\tau^+\nu$	$c\to s\bar{d}u$	$c\to se^-\bar{\nu}$
Γ	0.018	0.905	0.162

The color structure of the wave functions leads to the relations

$$\langle \Xi_{bc}^{\diamond}|(\bar{c}_iT_\mu c_k)(\bar{q}_k\gamma^\mu(1-\gamma_5)q_i)|\Xi_{bc}^{\diamond}\rangle = -\langle \Xi_{bc}^{\diamond}|(\bar{c}T_\mu c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bc}^{\diamond}\rangle,$$

where T_μ is an arbitrary spinor matrix.

4 Numerical estimates

Calculating the inclusive widths of decays for the Ξ_{bc}^{+} and Ξ_{bc}^{0} baryons, we have used the following set of dimensional parameters in the model:

$$m_c = 1.6, \quad m_l = 0.3, \quad m_b = m_c + 3.5, \quad m_s = m_l + 0.15, \quad \mu = 1.2, \quad T = 0.4, \quad (63)$$

where all numbers are in GeV. The baryon mass has been put to 7 GeV, and the wave function of the light constituent quark in the quark–diquark system has been taken in accordance to the relation

$$f = \sqrt{\frac{12}{M}}\Psi(0),$$

so that in the D-meson we would have $f_D = 200\,\text{MeV}$. For the wave function of the diquark subsystem, we have used the estimates in the non-relativistic model with the Buchmüller–Tye potential [21] with the color factor of the diquark. So,

$$\Psi^d(0) = 0.193\,\text{GeV}^{3/2}.$$

Further, it is quite evident that the estimates of the spectator widths of the free heavy quarks do not depend on the system wherein they are bound, so that we can exploit the results of the calculations performed earlier. We have chosen the quark masses to be the same as in [3], and we have put the corresponding values, presented in Table 1, as they stand in the paper mentioned.

Then the procedure, described above with the shown parameters, leads to the lifetimes of the Ξ_{bc}^{+} and Ξ_{bc}^{0} baryons:

$$\tau_{\Xi_{bc}^{+}} = 0.33\,\text{ps}, \quad (64)$$
$$\tau_{\Xi_{bc}^{0}} = 0.28\,\text{ps}. \quad (65)$$

We can clearly see that the difference in the lifetimes caused by the decay processes with the Pauli interference

Table 2. The branching fractions (in %) of the various modes in the inclusive decays of the Ξ_{bc}^{+} and Ξ_{bc}^{0} baryons

Mode	Γ_b	Γ_c	$\Gamma_{\rm PI}$	$\Gamma_{\rm WS}$
Ξ_{bc}^{+}	20	37	23	20
Ξ_{bc}^{0}	17	31	21	31

Table 3. The branching ratios for the inclusive semileptonic widths of Ξ_{bc}^{+} and Ξ_{bc}^{0}, in %

Mode	$\Gamma_c^{e\nu}$	$\Gamma_b^{e\nu}$	$\Gamma_b^{\tau\nu}$
Ξ_{bc}^{+}	5.0	4.9	2.3
Ξ_{bc}^{0}	4.2	4.1	1.9

and weak scattering is about 20%. The relative contributions by various terms in the total width of the baryons under consideration are presented in Table 2.

Note, that the contributions by the Pauli interference and weak scattering, depending on the baryon contents, can be significant – up to 40–50%. The corrections due to the quark–gluon operators of dimension 5 are numerically very small. The most important are the corrections due to the operator of dimension 3, where the role of time dilation is essential.

For the semileptonic decays, whose relative fractions are presented in Table 3, the largest corrections appear in the decays of the b-quark because of Pauli interference, so that the corresponding widths practically increase twice. This leads to the result that the semileptonic widths of b- and c-quarks in the electron mode are equal to each other, whereas for the spectator decays, the width of the charmed quark is twice that of b.

As for the sign of terms, caused by the Pauli interference, it is basically determined by the leading contribution, coming from the interference for the charmed quark of the initial state with the charmed quark from the b-quark decay. In this way, the antisymmetric color structure of the baryon wave function leads to a positive sign for the Pauli interference.

Finally, concerning the uncertainties of the estimates presented, we note that they are mainly related to the following:

(1) the spectator width of charmed quark, where the error can reach 50%, reflecting the agreement of the theoretical evaluation with the lifetimes of the charmed hadrons, so that for the baryons under consideration this term produces an uncertainty of $\delta\Gamma/\Gamma \approx 10\%$;
(2) the effects of Pauli interference in the decays of the beauty quark and in its weak scattering off the charmed quark from the initial state, wherein we use the non-relativistic wave function, which, being model-dependent, can lead to an error estimated to be close to 30%, producing an uncertainty of $\delta\Gamma/\Gamma \approx 15\%$ in the total widths.

Thus, we estimate that the uncertainty in the predictions of the total widths for the Ξ_{bc}^{+} and Ξ_{bc}^{0} baryons is about 25%.

5 Conclusion

In the present paper we have calculated the total lifetimes of baryons with two heavy quarks of different flavors: Ξ_{bc}^{+} and Ξ_{bc}^{0}, in the framework of an operator product expansion using the inverse heavy quark mass technique. In this way, we have taken into account the QCD corrections to the Wilson coefficients of the operators as well as the mass corrections. A peculiar role is played by the four-quark operators, responsible for the effects of Pauli interference and weak scattering off the constituents. These mechanisms are enforced due to the two-particle phase space in the final state, so that these effects, depending on the quark contents of the baryons, provide the contribution close to 50%. For the numerical estimates we get

$$\tau_{\Xi_{bc}^{+}} = 0.33 \pm 0.08\,\text{ps}, \tag{66}$$

$$\tau_{\Xi_{bc}^{0}} = 0.28 \pm 0.07\,\text{ps}. \tag{67}$$

We have also presented both the branching fractions of various contributions into the total widths and the inclusive semileptonic decays.

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